Rigorous Modal S-Matrix Analysis of the Cross–Iris in Rectangular Waveguides

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***Abstract*— A rigorous field theory analysis is presented for cross–irises in rectangular waveguides as well as for cross-iris coupled rectangular waveguide resonators. The theory is based on the full-wave mode-matching method for the key-building block discontinuity rectangular waveguide to crossed rectangular guide associated with the generalized S-matrix technique. Arbitrary iris location, unequal E- and H-plane cross size and finite thickness are rigorously taken into account. The scattering parameters of a cross-iris coupled one-resonator filter in the waveguide Ku-band (12–18 GHz) are presented as a calculation example. The theory is verified by comparison with available data for cut-off frequencies as well as with measurements.**

I. Introduction

CROSS-IRIS elements have found widespread applications in the design of waveguide dual-mode filters [1]–[3]. Although the modal characteristics of crossed rectangular waveguides are well known for a long time [4]–[6], in the design of cross-iris structures e.g., for filters, the workers in the field are still left to electric and magnetic polarizabilities [7] that are based on electrolytic tank measurements by Cohn [8]. However, when more stringent requirements are placed on the filters, or when additional post assembly fine tuning should be avoided, the need arises for more accurate design methods which allow to take into account both the finite thickness of the irises and the higher order mode interaction between them.

The purpose of this letter is to present a rigorous field-theory analysis of the cross-iris (Fig. 1) as well as for cross-iris coupled rectangular waveguide resonators (Fig. 4). The theory is based on the full-wave mode-matching method for the key-building block discontinuity rectangular waveguide to crossed rectangular guide associated with the generalized S-matrix technique. The combination with the already known key-building block asymmetrical rectangular waveguide double plane step [9] achieves the efficient analysis of arbitrary iris location, unequal E- and H-plane cross size and of the finite iris thickness. Moreover, composite structures, such as cross-iris coupled rectangular waveguide filters, may be rigorously taken into account. The efficiency of the method is demonstrated by designing a cross-iris coupled one-resonator filter in the waveguide Ku-band (12–18 GHz). The theory is verified by comparison with available data for cut-off frequencies as well as with measurements.

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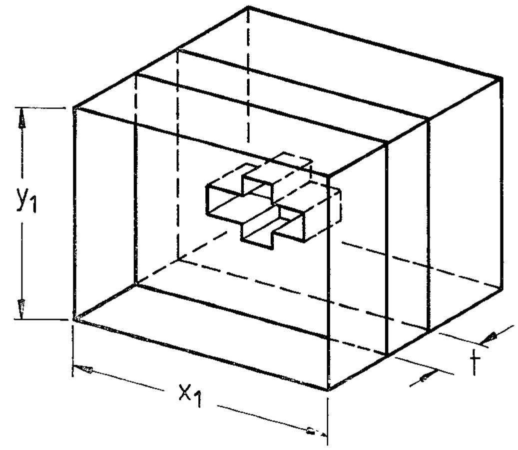

Fig. 1. Cross-iris with unequal side length and finite thickness.

II. Theory

The cross-iris (Fig. 1) is decomposed into two key building blocks: rectangular waveguide to crossed rectangular guide (Fig. 2(a)), and the double-step junction discontinuity (Fig. 2(b)). Combination with the modal scattering matrices of the corresponding intermediate homogeneous waveguide sections of finite lengths yields the total scattering matrix of the composed structure.

For the cross-section eigenvalue problem, electric and magnetic wall symmetry is utilized (Fig. 2(a)). Like in [10], the electromagnetic field in the subregions $i = I,\ II$

$$\vec{E}^i = \nabla \times \left(A^i_{Hz}\vec{e}_z\right) + \frac{1}{j\omega\epsilon}\nabla \times \nabla \times \left(A^i_{Ez}\vec{e}_z\right)$$
$$\vec{H}^i = \nabla \times \left(A^i_{Ez}\vec{e}_z\right) - \frac{1}{j\omega\mu}\nabla \times \nabla \times \left(A^i_{zHz}\vec{e}_z\right) \quad (1)$$

is derived from the z-components of two vector potentials

$$\vec{A}^i_{Hz} = \sum_{q=1}^{\infty}\left(\sqrt{Z^i_{Hq}}\right)\cdot T^i_{Hq}(x,y) \cdot \left[V^i_{Hq}\exp\left(-jk^i_{zHq}z\right) + R^i_{Hq}\exp\left(+jk_{zHq}z\right)\right]$$
$$\vec{A}^i_{Ez} = \sum_{p=1}^{\infty}\left(\sqrt{Y^i_{Ep}}\right)\cdot T^i_{Ep}(x,y) \cdot \left[V^i_{Ep}\exp\left(-jk^i_{zEp}z\right) - R^i_{Ep}\exp\left(+jk_{zEp}z\right)\right], \quad (2)$$

with the wave impedances

$$Z^i_{Hq} = (\omega\mu_0)/\left(k^i_{zHq}\right) = 1/Y^i_{Hq},$$
$$Y^i_{Ep} = (\omega\epsilon_0)/\left(k^i_{zEq}\right) = 1/Z^i_{Ep}. \quad (3)$$

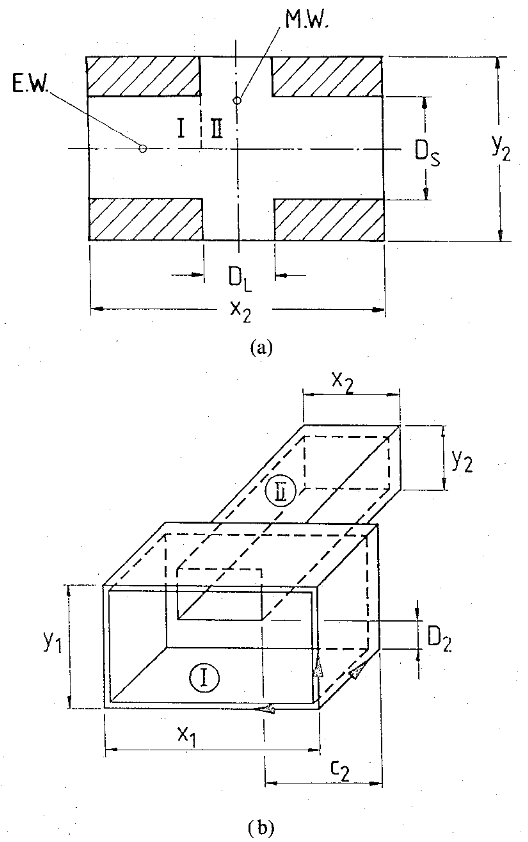

Fig. 2. Key building blocks for the modal S-matrix method. a) Rectangular waveguide to crossed rectangular guide. b) Double-step junction discontinuity [9].

$V^i_{H,E}$, $R^i_{H,E}$ are the TE- and TM-mode wave amplitudes of the forward and backward waves, respectively, which have to be related to each other at the corresponding discontinuity. This will yield the corresponding scattering matrix relations. k_z are the propagation factors, and T^i_{Hq}, T^i_{Ep} are the cross-section eigenfunctions of the corresponding waveguide structures under consideration, i.e., crossed guide, and rectangular waveguide. For the eigenvalue problem, the transverse resonance method is used [10]. This procedure reduces the size of the characteristic matrix equation to a quarter of the original size.

In order to calculate the modal scattering matrix of the key-building block discontinuity (Fig. 2(a)) directly by the corresponding field matching relations of the wave amplitude coefficients according to (2), the cross-section eigenfunctions are suitably normalized [10].

Matching the tangential field components of regions I and II at the common interface (Fig. 2(b)) yields the modal scattering matrix of the step discontinuity waveguide to crossed waveguide:

$$\begin{bmatrix} (R^I) \\ (V^{II}) \end{bmatrix} = (S) \begin{bmatrix} (V^I) \\ (R^{II}) \end{bmatrix}. \tag{4}$$

The modal scattering matrix of the rectangular waveguide double–step discontinuity is already given in [9]. The series of step discontinuities, for a complete crossed iris or filter structure, is calculated by direct combination of the single modal scattering matrices [9], [10].

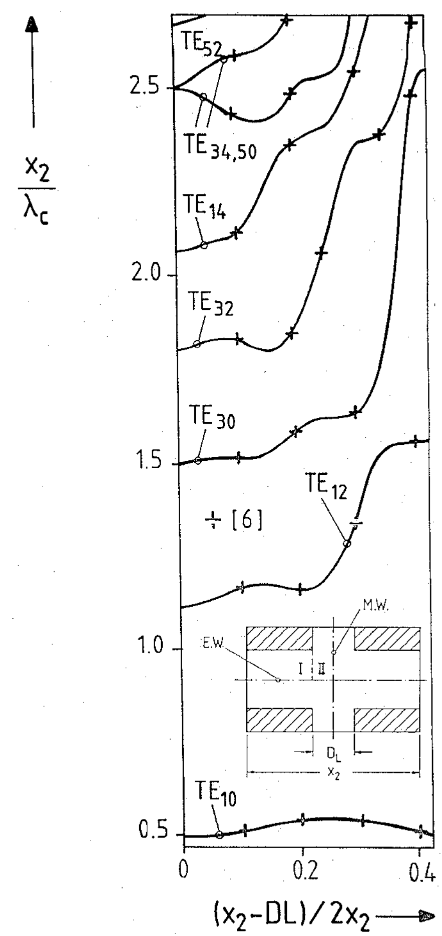

Fig. 3. Normalized modal cutoff-frequencies (x_2/λ_c) for TE magnetic modes of a symmetrical crossed rectangular waveguide ($x_2 = y_2$, $DL = DS$) as a function of $(x_2 - DL)/2x_2$, for TE_{10} excitation, as an example. Results are compared with those rep [6].

The number of TE-modes, TM-modes, and cross-sectional expansion terms [10] are chosen to be 12, 7, 15 for the optimization, and 20, 14, 20 for the final analysis, respectively. For better convergence, the numbers of the expansion terms in the subregions I and II (Fig. 2(a)) are chosen so that the values relate to the ratio of the corresponding y dimensions, i.e., for the final analysis, e.g., 10:20 if the ratio of the y dimensions is 1:2. The choice of the numbers of the modes and expansion terms has been verified by checking the convergence behavior against the already available cutoff frequencies [6] as well as against the results of own measurements.

III. Results

In order to verify the theory for the cross-section eigenvalue problem, Fig. 3 presents the normalized modal cutoff-frequencies (x_2/λ_c) for the TE magnetic modes of a symmetrical crossed rectangular waveguide as a function of $(x_2 - DL)/2x_2$, for TE_{10} excitation, as an example. The results are compared with those reported in [6]. Excellent agreement may be stated.

Fig. 4 shows the insertion and return loss versus frequency of a cross-iris coupled one-resonator filter designed for a midband frequency of about 15 GHz in the waveguide Ku-

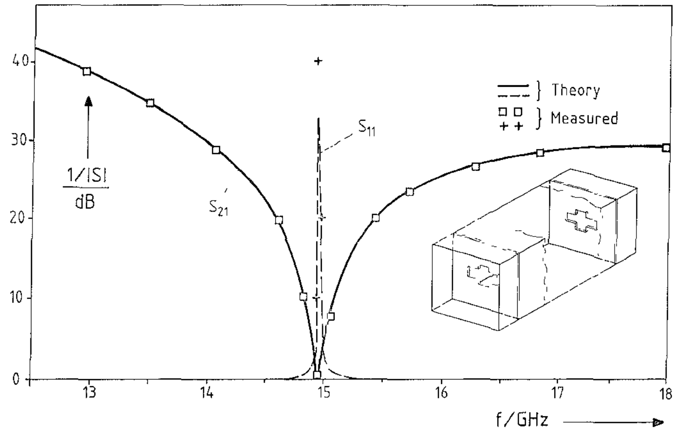

Fig. 4. Insertion and return loss versus frequency of a cross-iris coupled one-resonator filter. WR 62 waveguide housing: $x_1 = 15.799$ mm, $y_1 = 7.899$ mm. Iris dimensions: $x_2 = 4.42$ mm, $y_2 = 4.79$ mm, $DL = 3.08$ mm, $DS = 3.379$ mm, thickness $t = 0.21$ mm. Resonator length: 12.155 mm. Comparison with measurements ($+++, \square\square\square$).

band (WR 62 waveguide housing: 15.799 mm $\times$ 7.899 mm). The filter has been fabricated by using metal etching for the 210-μm thick cross-irises. The theory is verified by excellent agreement with the measurements.

IV. Conclusion

The rigorous modal scattering matrix method presented in this letter achieves the efficient and accurate analysis of cross-irises in rectangular waveguides as well as for cross-iris coupled rectangular waveguide resonators. Since the theory includes the finite thickness of the irises as well as the higher order mode interaction between them, all relevant design parameters may be rigorously taken into account. Therefore, excellent agreement with measured results may be obtained.

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